# ON REINFORCED INCOMPLETE BLOCK DESIGNS

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## Introduction

The problem of large intrablock variance in complete block designs involving a large number of treatments was, no doubt, solved by adopting incomplete block designs (Yates, 1936, 1937). But the solution could not ensure full freedom of action under all types of situations, as designs could not be available for each and every number of treatments keeping at the same time the total number of plots at any desired level. By adopting lattice designs the total number of plots can be kept at a lower level but such designs are available only for specified numbers of treatments. The partially balanced incomplete block designs introduced by Bose and Nair (1939) contributed further to solve the problem but the problem could not be eliminated. The author (Das, 1954) introduced a design obtainable by adding some extra treatments to each of the blocks of a balanced incomplete block design together with some extra blocks, if necessary, each of which contains all the treatments. This design actually solves the problem to a very great extent. With a given number of treatments $t$, the $a$ extra treatments can always be so adjusted that with $v\ (= t - a)$ treatments a balanced incomplete block design becomes available. Though such a design is available for any $t$, due to the limitations on $v$ for getting a B.I.B. design it may not always be possible to keep in such designs the total number of plot at any desired level having at the same time smaller blocks. Hence the necessity of building such designs around any of the available incomplete block designs so that the difficulty due to the limitation on $v$ in the previous design can be avoided by having the freedom of choosing an incomplete block design from a greater field of designs.

From a suggestion by the author Giri (1957) worked out the method of analysis of such a design built around the partially balanced incomplete block designs with two associate classes. According to a suggestion from K. R. Nair the design was called Reinforced Partially Balanced incomplete block designs, as such designs are obtainable by reinforcing the P.B.I.B. designs by means of some extra treatments and blocks. In this paper the method of analysis suitable for the general reinforced incomplete block designs, has been presented and this provides a solution of the main problem with the incomplete block designs of getting designs with any number of treatments keeping at the same time the total number of plots at a desired level. It has been pointed out to the author by K. R. Nair that the reinforced B.I.B. design with no complete blocks added is a particular case of the intra- and inter-group balanced incomplete block designs introduced by Nair and Rao (1942). The parameters of their design for the case of 2 groups when linked up with those of the reinfroced B.I.B. designs when $\beta = 0$ are as follows:—

$$\lambda_{11} = \lambda, \; \lambda_{12} = r, \; \lambda_{22} = b.$$

$$v_1 = v, \; v_2 = a, \; R = k + a, \; r_1 = r, \; r_2 = b.$$

*Definition of the design.*—Let $v$, $b$, $r$, $k$, etc., be the parameters of any incomplete block design. If $a \geqslant 1$ new treatments are now introduced and all included in each of the blocks of the incomplete block design together with $\beta \geqslant 0$ new blocks each having all the $v + a$ treatments in it, then the new design thus built around the incomplete block design can be called a reinforced incomplete block design with $t = v + a$ treatments, $b + \beta$ blocks of sizes $k + a$ and $v + a$ and with two types of replications, *viz.*, $r + \beta$ and $b + \beta$.

In the case of field experiments, designs with $\beta > 0$ are not suitable as these will then have two block sizes and therefore two intra-block errors, a situation requiring special treatments. In designs with animals as experimental units provision of designs with $\beta > 0$ often helps (Finney, 1952) and in such cases no special difficulty of analysis arises as it is very likely that the intra-litter variation is not dependent on the size of the litters (Finney, 1952).

For any given number of treatments $t$, $v$ and $a$ can always be so chosen that an incomplete block design with $v$ treatments and a desired number of replications is available and then by including each of the $a$ treatments in each of the blocks of the incomplete block design the desired design for the $t$ treatments can be obtained. The procedure thus ensures not only a desired number of overall experimental plots but also the availability of the design whatever the number of treatments.

While choosing $a$ it always pays to keep it as low as possible, as greater values of $a$ increase the block size. No general criterion can be set as to how to allot the different treatments into the two groups. But once the problem is known specifically, such allotment is not difficult. Thus, in Bio-assays the doses in a parallel line assay should be so divided that the two preparations are represented equally in each of the groups. Again, if it is necessary to estimate the regression contrast with greater precision, the doses should be so allotted to the groups that the numbers of observations against the doses involved in the contrast are greater. If the number of replications available for some of the treatments be lmited as in progeny row trials (Hutchinson and Panse, 1937), the allotment should be made on the basis of the number of replications so that the maximum number of replications can be accommodated.

## Method of Analysis

On the usual model that $y_{ij} = \mu + t_i + b_j + \epsilon_{ij}$, where $\mu$ is the grand mean, $t_i$ and $b_j$ are respectively the effects of the $i$-th treatment and the $j$-th block and $\epsilon_{ij}$ a random variable with zero mean and a constant variance, $\sigma^2$, the normal equations for estimating the treatment effects after eliminating the block effects come out, when written according to the method given by the author (Das, 1953) after eliminating one of the $\alpha$ treatments, as

$$(b + \beta)\, t_i + \frac{b - r}{k + \alpha} \sum t_m = Q_i \; (i = 1,\ 2, \ldots \alpha) \tag{1}$$

$$(r + \beta)\, t_m + \sum_{m' \neq m} \frac{(r - \lambda_{mm'})}{k + \alpha}\, t_{m'} = Q_m \; (m = 1,\ 2,\ \ldots v) \tag{2}$$

Alternatively, equation (2) can be written as

$$\left(r + \beta - \frac{r}{k + \alpha}\right) t_m - \frac{1}{k + \alpha} \sum_{m' \neq m} \lambda_{mm'}\, t_{m'} = Q_m - \frac{r\, \Sigma\, t_m}{k + \alpha} \tag{3}$$

where $t_i$ stands for the extra $\alpha$ treatments, $t_m$ for the $v$ treatments involved in the original incomplete block design, $\lambda_{mm'}$ is the number of times the treatments $m$ and $m'$ occur together in the same block and $Q_i$ and $Q_m$ are the adjusted total yields for the $i$-th and the $m$-th treatments respectively.

By adding equations (2) over all the $v$ treatments we get:

$$(r + \beta) \sum t_m + \frac{k\,(b - r)}{k + \alpha} \sum t_m = \sum Q_m$$

Hence,

$$\sum t_m = \frac{(k + \alpha)\, \Sigma\, Q_m}{k\,(b + \beta) + \alpha\,(r + \beta)} \tag{4}$$

Thus, substituting (4) in (1) the solution for $t_i$ can be obtained.

From equation (3) it is seen that the normal equations in the original incomplete block designs with only $v$ treatments, are of the same form as what has been obtained when the design has been reinforced, the only differences being that $r - (r/k)$ has been replaced by $r + \beta - (r/k + a,)$, $Q_m$ by $Q_m - r\Sigma t_m/k + a$ and $\Sigma t_m = 0$ by $\Sigma t_m =$ a known quantitiy. As $\Sigma t_m$ is known, the solution for (3), *i.e.*, of $t_m$ can easily be obtained from the solution of $t_m$ in the original design by taking into account the changes in the equations mentioned earlier.

The adjusted treatment S.S. and the error S.S. can now be obtained in the usual manner.

In agricultural experiments where $\beta = 0$ it is seen that the normal equations and consequently their solution for $t_m$ in the reinforced design, remains the same as in the original design except for the fact that $k$ is to be replaced by $k + a$. Hence the expression for the variance of $(t_m - t_{m'})$ in such designs can be obtained from that of the original design by replacing $k$ by $k + a$. The variances of $(t_i - t_m)$ as also of $(t_i - t_{i'})$ can be obtained from the solution for $t_i$ and $t_m$.

Coming to the efficiency factor of the reinforced design for comparisons of the form $(t_m - t_{m'})$ it appears that the variances of such contrasts are decreasing functions of the block size. Though the general proof of this has not yet been available, it has been shown by the author that it is true in the case of P.B.I.B. designs with two associate classes and the proof has been included in the paper of Giri (1957).

To work out the combined estimate after recovering the inter-block information it is found that no such information exists for the set of $a$ treatments. The combined estimate of the other $v$ treatments can be obtained as in the original design by making those changes in the coefficients of the equations which have been found necessary in the case of intra-block analysis.

## Summary

A generalised reinforced incomplete block design has been defined. It has been shown that the design enjoys so much of flexibility that such designs become available to meet almost all situations. Both inter- and intra-block analyses have been worked out in the paper.

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